EXERCISE – I HINTS & SOLUTIONS

Sol.1 A

$$\int_1^x \frac{dt}{|t|\sqrt{t^2-1}} = \frac{\pi}{6}$$

$$\sec^{-1}t\,\Big|_1^x = \frac{\pi}{6}$$

$$\sec^{-1}x = \frac{\pi}{6} \Rightarrow x = \sec\frac{\pi}{6} \Rightarrow x = \frac{2}{\sqrt{3}}$$

Sol.2 C

$$f(x) = \begin{cases} x : x < 1 \\ x-1 : x \ge 1 \end{cases}$$

$$I = \int_0^2 x^2 f(x)dx = \int_0^1 x^2 f(x)dx + \int_1^2 x^2 f(x)dx$$

$$= \int_1^1 x^3 dx + \int_1^2 (x^3 - x^2)dx = \frac{5}{3}$$

Sol.3 C

$$\int_n^{n+1} f(x)dx = n^2$$

$$\int_{-2}^4 f(x)dx = \int_{-2}^{-1} f(x)dx + \int_{-1}^0 f(x)dx + \int_0^1 f(x)dx$$

$$+ \int_1^2 f(x)dx + \int_2^3 f(x)dx + \int_3^4 f(x)dx$$

= 4 + 1 + 0 + 1 + 4 + 9 = 19

Sol.4 D

$$I = \int_0^\pi |1+2\cos x|\,dx$$

$$= \int_0^{2\pi/3} (1+2\cos x)dx - \int_{2\pi/3}^{\pi} (1+2\cos x)dx$$

$$= (x+2\sec x)\Big|_0^{2\pi/3} - (x+2\sec x)\Big|_{2\pi/3}^{x}$$

$$= \frac{\pi}{3} + 2\sqrt{3}$$

Sol.5 A

$$I = \int_{-1}^3 (|x-2|+[x])dx$$

$$= \int_{-1}^2 |x-2|+dx + \int_2^3 |x-2|+dx + \int_{-1}^0 (-1)dx$$

$$+ \int_0^1 0.dx + \int_1^2 1.dx + \int_2^3 2.dx$$

$$= \int_{-1}^2 (2-x)\,dx + \int_2^3 (x-2)dx \;-1 + 0 + 1 + 2$$

$$= 2x - \frac{x^2}{2}\Bigg|_{-1}^{2} + \frac{x^2-2x}{2}\Bigg|_2^3 + 2 = 7$$

Sol.6 C

$$I = \int_{\ell n\lambda}^{-\ell n\lambda} \frac{f\left(\frac{x^2}{4}\right)[f(x)-f(-x)]}{f\left(\frac{x^2}{4}\right)[g(x)+g(-x)]}dx$$

odd function by P – 5

I = 0

Sol.7 A

$$\int_{-1}^{3/2} |x\sin\pi x|\,dx$$

$$= \int_{-1}^{1} |x\sin\pi x|\,dx + \int_1^{3/2} |x\sin\pi x|\,dx$$

$$= 2\int_0^1 |x\sin\pi x|\,dx + \int_1^{3/2} |x\sin\pi x|\,dx$$

$$= 2\int_0^1 x\sin\pi x\,dx - \int_1^{3/2} x\sin\pi x\,dx$$

$$\int x\cos\pi x dx = -\frac{x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2}$$

$$\int_0^1 x\sin \pi x dx = \frac{1}{\pi}$$

$$\int_1^{3/2} x\sin \pi x\, dx = \frac{-1}{\pi^2} - \frac{1}{\pi}$$

$$I = \frac{2}{\pi} + \frac{2}{\pi^2} + \frac{1}{\pi} = \frac{3\pi+1}{\pi^2}$$

$k = 3\pi + 1$

Sol.8 B

$$I = \int_0^{\pi/4} \frac{x\sin x}{\cos^3 x} dx = \int_0^{\pi/4} x\tan x \sec^2 x\, dx$$

$$= x\left.\frac{\tan^2 x}{2}\right|_0^{\pi/4} - \int \frac{\tan^2 x}{2} dx$$

$$= \frac{\pi}{8} - \frac{1}{2}\int (\sec^2 x - 1)\, dx$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[\tan x - x\right]_0^{\pi/4} = \frac{\pi}{4} - \frac{1}{2}$$

Sol.9 C

$f(0) = 1, f(2) = 3, f'(2) = 5, f'(0)$ is finite

$$I = \int_0^1 x f''(2x)\, dx = \left.\frac{x f'(2x)}{2}\right|_0^1 - \frac{1}{2}\int_0^1 f'(2x)\, dx$$

$$= \frac{f'(2)}{2} - \frac{1}{4} f(2x)|_0^1 \qquad = \frac{5}{2} - \frac{1}{4}[f(2) - f(0)] = 2$$

Sol.10 A

$$I = \int_{\log\pi - \log 2}^{\log\pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)} dx$$

Put $\dfrac{e^x}{3} = t \Rightarrow e^x\, dx = 3dt$

$$= 3\int_{\pi/6}^{\pi/3} \frac{dt}{1-\cos 2t} = \frac{3}{2}\int_{\pi/4}^{\pi/3} \frac{dt}{\sin^2 t} = \frac{3}{2}\int \operatorname{cosec}^2 t\, dt$$

$$= -\frac{3}{2}[\cot t]_{\pi/6}^{\pi/3} = -\frac{3}{2}\left[\frac{1}{\sqrt{3}} - \sqrt{3}\right] = \sqrt{3}$$

Sol.11 A

$$I_1 = \int_e^{e^2} \frac{dx}{\ell nx} \quad ; \quad I_2 = \int_1^2 \frac{e}{x} dx$$

Put $\ell nx = t \Rightarrow x = e^t \Rightarrow dx = e^t dt$

$$I_1 = \int_1^2 \frac{e^t}{x} dt = I_2$$

Sol.12 D

$$2I = \int_{2-\log 3}^{3+\log 3} \frac{\log(4+x)}{\log(4+x) + \log(9-x)} dx$$

using king

replace x by 5 – x

$$I = \int_{2-\log 3}^{3+\log 3} \frac{\log(9-x)}{\log(9-x) + \log(4+x)} dx$$

$$2I = \int_{2-\log 3}^{3+\log 3} dx \Rightarrow I = \frac{1}{2} + \log 3$$

Sol.13 C

$$I_1 = \int_0^{3\pi} f(\cos^2 x) dx \quad ; \quad \text{period is } \pi$$

$$= 3\int_0^{\pi} f(\cos^2 x) dx$$

$I_1 = 3I_3$

Similalry $\quad I_2 = 2I_3$

$I_2 + I_3 \;\; 3I_3$

$I_2 + I_3 = I_1$

Sol.14 C

$$I = \int_0^{11} \frac{11^x}{11^{[x]}} dx = \frac{k}{\log 11}$$

$$I = \int_0^{11} \frac{11^x}{11^{[x]}} dx = \int_0^{11} \frac{11^x}{11^{x-\{x\}}} dx$$

$$= \int_0^{11} 11^{\{x\}} dx = 11\int_0^1 11^x dx = \frac{11}{\log 11}[11^x]_0^1 = \frac{110}{\log 11}$$

k = 110

Sol.15 D

$$f(x) = 1 + x + \int_1^x (\ell n^2 t + 2\ell n t)\,dt$$

$f'(x) = 1 + \ell n^2 x + 2\ell n x$

$f'(x) = 0$

$(\ell n x + 1)^2 = 0 \Rightarrow x = e^{-1}$

$$f\left(\frac{1}{e}\right) = 1 + \frac{1}{e} + \int_1^{1/e}\left[\underset{\substack{\uparrow \\ f(t)}}{\ell n^2 t} + \underset{\substack{\uparrow \\ f'(t)}}{\left(\frac{2}{t}\ell n t\right)}\right]dt$$

$$= 1 + \frac{1}{e} + t\ \ell n^2 t\,\Big|_1^{1/e} = 1 + \frac{2}{e} = 1 + 2e^{-1}$$

Sol.16 B

$$\int_a^y \cos t^2\,dt = \int_a^{x^2} \frac{\sin t}{t}\,dt$$

diff. w.r.t. x both side

$$\cos y^2 \frac{dy}{dx} = \frac{\sin x^2}{x^2} \,.\, 2x$$

$$\frac{dy}{dx} = \frac{2\sin x^2}{x\cos y^2}$$

Sol.17 D

$$I = \lim_{n\to\infty} \sum_{r=1}^{n}\left(\frac{r^3}{r^4 + n^4}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n}\left(\frac{r^3}{n^4\left(1+\frac{r}{n}\right)^4}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n}\left(\frac{\left(\frac{r}{n}\right)^3}{1+\left(\frac{r}{n}\right)^4}\,\frac{1}{n}\right) = \int_0^1 \frac{x^3}{1+x^4}\,dx$$

$$= \frac{1}{4}\ \ell n\ (1+x^4)\,\Big|_0^1 = \frac{1}{4}\ \ell n\ 2$$

Sol.18 B

$$\lim_{n\to\infty} \sum_{r=2n+1}^{3n} \frac{n}{r^2 - n^2}$$

$$= \lim_{n\to\infty} \sum_{r=2n+1}^{3n} \frac{1}{\left(\left(\frac{r}{n}\right)^2 - 1\right)} \cdot \frac{1}{n}$$

$$= \int_2^3 \frac{dx}{x^2-1} = \frac{1}{2}\ \ell n\ \frac{x-1}{x+1}\Big|_2^3 = \ell n\ \sqrt{\frac{3}{2}}$$

Sol.19 C

$$L = \lim_{h\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{h^2}\right)\ldots\left(1+\frac{n^2}{h^2}\right)\right]^{1/n}$$

$$\ell n L = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \ell n\left(1+\left(\frac{r}{n}\right)^2\right)$$

$$= \int_0^1 \ell n(1+x^2)\,dx$$

$$= x\ell n\ (1 + x^2) - 2x + 2\tan^{-1} x\,\Big|_0^1$$

$$\ell n L = \ell n 2 - 2 + \frac{\pi}{2} \Rightarrow L = \frac{2}{e^2}\ e^{\pi/2}$$

Sol.20 C

$$I = \lim_{n\to\infty} \frac{\pi}{n}\left[\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \ldots + \sin\frac{(n-1)\pi}{n}\right]$$

$$= \lim_{n\to\infty} \frac{\pi}{n}\sum_{r=1}^{n-1} \sin\left(\frac{r\pi}{n}\right) = \pi\int_0^1 \sin(\pi x)\,dx$$

$$= -\frac{\pi}{\pi}\ [\cos \pi x]_0^1 = 2$$

Sol.21 D

$$f(x) = \frac{-1}{x^2} f\left(\frac{1}{x}\right)$$

$$I = \int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)\,dx = -\int_{\sin\theta}^{\operatorname{cosec}\theta} \frac{1}{x^2} f\left(\frac{1}{x}\right) dx$$

$$\frac{1}{x} = t \Rightarrow -\frac{1}{x^2}dx = dt$$

$$I = \int_{\operatorname{cosec}\theta}^{\sin\theta} f(t)\,dt = -\int_{\sin\theta}^{\operatorname{cosec}\theta} f(t)\,dt$$

$$2I = 0 \Rightarrow I = 0$$

Sol.22 D

$$I = \int_0^{\left(\frac{\pi}{2}\right)^{1/3}} x^5 \sin x^3 dx$$

$$x^3 = t \Rightarrow x^2 dx = \frac{dt}{3}$$

$$I = \frac{1}{3}\int_0^{\pi/2} t \sin t\,dt = \frac{1}{3}\left[-t\cos t + \int \cos t\,dt\right]$$

$$= \frac{1}{3}[-t\cos t + \sin t]_0^{\pi/2} = \frac{1}{3}$$

Sol.23 A

$$L = \lim_{n\to\infty}\left(\sin\frac{\pi}{2n}\sin\frac{2\pi}{2n}.\sin\frac{3\pi}{2n}...\sin\frac{(n-1)\pi}{2n}\right)^{1/n}$$

$$\ell n L = \lim_{h\to\infty}\frac{1}{n}\left[\ell n\sin\frac{\pi}{2n} + \ell n\sin\frac{2\pi}{2n} + ... + \ell n\frac{\sin(n-1)\pi}{2n}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n-1}\ell n\sin\frac{r\pi}{2n}$$

$$\ell n L = \int_0^1 \ell n \sin\frac{\pi x}{2}dx$$

$$\frac{\pi x}{2} = t \Rightarrow dx = \frac{2}{\pi}dt$$

$$= \frac{2}{\pi}\int_0^{\pi/2}\ell n\sin t\,dt = \frac{2}{\pi}\left[-\frac{\pi}{2}\ell n 2\right]$$

$$\ell n L = \ell n\frac{1}{2} \Rightarrow L = \frac{1}{2}$$

Sol.24 B

$f(x) = f(a-x)$

$g(x) + g(a-x) = 2$

$$I = \int_0^a f(x)\,g(x)\,dx = \int_0^a f(a-x)\,g(a-x)\,dx$$

$$I = \int_0^a f(x)\,(2 - g(x))\,dx = 2\int_0^a f(x)\,dx - I$$

$$2I = 2\int_0^a f(x)\,dx \Rightarrow I = \int_0^a f(x)\,dx$$

Sol.25 C

$$I = \int_4^{10}\frac{[x^2]}{[(x-14)]^2 + [x^2]}dx$$

$$I = \int_4^{10}\frac{[x-14]^2}{[(x-14)^2] + [x^2]}dx$$

$$2I = \int_4^{10} 1.dx \Rightarrow I = 3$$

Sol.26 B

$$I = \int_0^{\infty}[2e^{-x}]\,dx$$

Let $2e^{-x} = t$

$$-2e^{-x}dx = dt \Rightarrow dx = \frac{-dt}{t}$$

$$= -\int_2^0 [t]\frac{dt}{t} = \int_0^2 [t]\frac{dt}{t} = \int_0^1 \frac{0}{t}dt + \int_1^2 \frac{1}{t}dt = \ell n 2$$

Sol.27 B

$$I = \sum_{r=1}^{100}\left(\int_0^1 f(r-1+x)\,dx\right)$$

$$= \int_0^1 f(x)\,dx + \int_0^1 f(1+x)\,dx +$$

$$\int_0^1 f(2+x)\,dx + ... + \int_0^1 f(99+x)\,dx$$

$$= \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx + ... + \int_{99}^{100} f(x)\,dx$$

$$= \int_0^{100} f(x)\,dx = 1 = a$$

Sol.28 B

$$f(x) = \int_0^x \sin[2x]\,dx$$

$$f\left(\frac{\pi}{2}\right) = \int_0^{\pi/2} \sin[2x]\,dx$$

Put $2x = t \Rightarrow dx = \frac{dt}{2} = \frac{1}{2}\int_0^{\pi} \sin[t]\,dt$

$$= \frac{1}{2}\left[\int_0^1 \sin 0\,dt + \int_1^2 \sin 1.dt + \int_2^3 \sin 2.dt + \int_3^x \sin 3.dt\right]$$

$$= \frac{1}{2}\,[\sin 1 + \sin 2 + (\pi - 3)\sin 3]$$

Sol.29 A

$$A = \int_0^{\pi} \frac{\cos x}{(x+2)^2}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sin 2x}{(x+1)}\,dx$$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

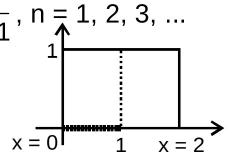
$$= \frac{1}{2}\int_0^{\pi} \frac{\sin t\,dt}{\left(\frac{t}{2}+1\right)} = \int_0^{\pi} \frac{\sin t\,dt}{(t+2)} = \left.\frac{-\cot}{t+2}\right|_0^{\pi} - \int_0^{\pi} \frac{\sin t}{(t+2)^2}\,dt$$

$$I = \frac{1}{2} + \frac{1}{\pi+2} - A$$

Sol.30 C

f(x) $= 0$ where $\pi = \frac{n}{n+1}$, n = 1, 2, 3, ...

$= 1$ else where



Find $= \int_0^2 f(x)dx = 2 \times 1 = 2$

Sol.31 B

$$I = \int_{-\pi/2}^{\pi/2} \frac{|x|\,dx}{8\cos^2 2x + 1} = 2\int_0^{\pi/2} \frac{|x|\,dx}{8\cos^2 2x+1}$$

$$I = 2\int_0^{\pi/2} \frac{x\,dx}{8\cos^2 2x + 1}$$

$$I = 2\int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right)dx}{8\cos^2 2x+1}$$

$$2I = \pi \int_0^{\pi/2} \frac{dx}{8\cos^2 2x+1}$$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

$$2I = \frac{\pi}{2}\int_0^{\pi} \frac{dt}{8\cos^2 t+1}$$

$$2I = \pi \int_{\pi/2}^{\pi/2} \frac{-dt}{8\cos^2 t+1}$$

$$I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sec^2 t\,dt}{a+\tan^2 t} = \frac{3}{2}\left(\frac{1}{3}\right)\tan^{-1}\tan t\Big|_0^{\pi/2} = \frac{\pi^2}{12}$$

Sol.32 D

$$\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{x}}{2}$$

$\int_0^{\infty} e^{-ax^2}dx$ Put $\sqrt{a}x = t$

$$dx = \frac{dt}{\sqrt{a}} = \frac{1}{\sqrt{a}}\int_0^{\infty} e^{-t^2}\,dt = \frac{\sqrt{3}}{2\sqrt{a}} = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

Sol.33 D

$$I = \frac{\int_0^n [x]dx}{\int_0^n \{x\}dx} = \frac{\int_0^n x - \{x\}dx}{\int_0^n \{x\}dx} = \frac{\int_0^n x dx}{\int_0^n \{x\}dx} = 1$$

$$= \frac{\frac{n^2}{2}}{n\int_0^1 \{x\}dx} - 1 = \frac{\pi^2/2}{n.\frac{1}{2}} - 1 = n - 1$$

Sol.34 B

$$A = \int_0^1 \frac{e^t dt}{1+t}$$

$$I = \int_{a-1}^{a} \frac{e^{-t}}{t-a-1} dt = -\int_{a-1}^{a} \frac{e^{-t}}{1+a-t} dt$$

Put $a - t = z$

$dt = - dz$

$$= \int_1^0 \frac{e^{a(z-a)}dz}{1+z} = -\int_0^1 \frac{e^z.dz}{1+z} = -Ae^{-a}$$

Sol.35 D

$$I = \int_0^{2n\pi} |\sin x|_{sec} - \int_0^{2n\pi} \left[\left|\frac{\sin x}{2}\right|\right]_{dx}^{a}$$

$$= \int_0^{2n\pi} |\sin x| dx \quad -1 \le \sin x \le 1$$

$$= 2n \int_0^{\pi} |\sin x| dx \quad -\frac{1}{2} \le \frac{\sin x}{2} \le \frac{1}{2}$$

$$= 2n\,(2) = 4n \frac{A}{2} \quad \left[\left|\frac{\sin x}{2}\right|\right] \to 0$$

Sol.36 D

$$\int_0^{\pi/3} f(x)dx = 0 \int_0^{\pi/4} \tan x dx + \int_{\pi/4}^{\pi/3} \cot x\, dx$$

$$= \ell n \sec x |_0^{\pi/4} + \ell n \sec x |_{\pi/4}^{\pi/3}$$

$$= \ell n \sqrt{2} + \ell n \frac{\sqrt{3}}{2} - \ell n \frac{1}{\sqrt{2}} = \ell n \sqrt{3}$$

Sol.37 C

$$I = \int_1^2 ([x^2] - [x]^2) dx$$

$$= \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2.dx + \int_{\sqrt{3}}^{2} 3.dx - \int_1^2 1.dx$$

$$= (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3\,(2 - \sqrt{3}) - 1$$

$$= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3} - 1$$

$$4 - \sqrt{2} - \sqrt{3}$$

Sol.38 B

$$I = \int_0^{\pi} f(x)\sin x\, dx + \int_0^{\pi} f''(x)\sin x\, dx$$

$$= -\cos x + f(x)\,|_0^{\pi} + \int_0^{\pi} f'(x)\cos x\, dx$$

$$+ \sin x\, f'(x)|_0^{\pi} - \int_0^{\pi} \cos f'(x)\, dx$$

$I = f(x) + f(0)$

$5 = f(x) + f(0)$

$5 = 2 + f(0)$

$f(0) = 3$

Sol.39 C

$$U_n = \int_0^{n/2} x^n.\sin x dx$$

$$U_{10} = \int_0^{x/2} x^{10}.\sin x dx$$

$$= -x^{10} \cos x |_0^{x/2} + \int_0^{x/2} 10x^9 \cos dx$$

$$= -\left(\frac{\pi}{2}\right)^{10} . \cos \frac{\pi}{2} + 10 \int_0^{x/2} x^9 \cos x dx$$

$$= 10\,[x^9 \sin x |_0^{x/2} - 9 \quad 9 \int_0^{x/2} x^8 \sin x dx$$

$$U_{10} = \left(\frac{\pi}{2}\right)^9 - 90\, U_8$$

$$U_{10} + 90\, U_8 = 10\left(\frac{\pi}{2}\right)^9$$

Sol.40 A

$f(x) = e^{g(x)}$

$$g(x) = \int_2^x \frac{t\, dt}{1+t^4}$$

$$g'(x) = \frac{x}{1+x^4}$$

$g'(2) = \frac{2}{17}$

$f'(x) = e^{g(x)} . g'(x)$

$f'(2) = e^{g(x)} . g'(2)$

$= e^0 . \frac{2}{17} = \frac{2}{17}$

Sol.41 A

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{r}{n}\right)\sec^2\left(\frac{r}{n}\right)^2 = \int_0^1 x\sec^2 x^2\,dx$$

Put $x^2 = t$

$$x\,dx = \frac{dt}{2} = \frac{1}{2}\int_0^1 \sec^2 t = \frac{1}{2}\,[\tan t]_0^{\,1} = \frac{1}{2}\tan 1$$

Sol.42 A

$$\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{r}{n}\right)^p . \frac{1}{n}$$

$$\int_0^1 x^p dx = \left.\frac{x^{p+1}}{p+1}\right|_0^1 = \frac{1}{p+1}$$

Sol.43 A

$$f(x) = f(x) + f\left(\frac{1}{x}\right)$$

$$f(x) = \int_4^x \frac{\log t}{1+t}dt$$

$$f\left(\frac{1}{x}\right) = \int_4^{1/x} \frac{\log t}{1+t}dt$$

$$t = \frac{1}{4} \Rightarrow dt = -\frac{du}{u^2}$$

$$= -\int_4^u \frac{\ell n - \frac{1}{4}}{1+\frac{1}{4}} . \frac{du}{u^2} = \int_1^x \frac{\ell n 4}{u(1+u)}du$$

$$f\left(\frac{1}{x}\right) \int_1^x \frac{\ell nt}{(1+t)t}dt$$

$$f(x) = f(x) + f\left(\frac{1}{x}\right) = \int_1^x \frac{\ln t}{(t+1)}\left(\frac{1+1}{t}\right)dt$$

$$f(x) = \int_1^x \frac{\ell nt}{t}dt = \left.\frac{\ell n^2 t}{2}\right|_1^x$$

$$f(x) = \frac{\ell n^2 x}{2} \Rightarrow f(e) = \frac{\ell n^2 e}{2} = \frac{1}{2}$$

Sol.44 B

$$I = \int_{-3\pi/2}^{-\pi/2} \{(x+x)^3 + \cos^2(x+3x)\}\,dx$$

Put $x + x = t \Rightarrow dx = d\,t$

$$I = \int_{-\pi/2}^{\pi/2} (t^3 + \cos^2 t)\,dt$$

$$I = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$$

Sol.45 A

$$\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}}\ d = \frac{\pi}{2}$$

$$\sec^{-1}x - \sec^{-1}\sqrt{2} = \frac{\pi}{2}$$

$$\sec^{-1}x = \frac{\pi}{2} = \frac{\pi}{2}$$

$$\sec^{-1}x = \frac{\pi}{2} + \frac{\pi}{4}$$

$$\sec^{-1}x = \frac{3\pi}{4}$$

$$x = \sec\frac{3\pi}{4}$$

$$x = -\sqrt{2}$$

Sol.46 C

$$I = \int_0^{\pi} x f(\sec x)\,dx$$

King and add

$$2I = \pi\int_0^{\pi} x f(\sec x)\,dx$$

$$I = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$$

$$I = \pi \int_0^{\pi} f(\sin x)\,dx$$

$$I = \frac{\pi}{2}\int_0^{2} f(\sin x)\,dx$$

$$I = \pi \int_0^{\pi/2} f(\sin x)\,dx = \pi \int_0^{\pi/2} f(\cos x)\,dx$$

Sol.47 A

$$I = \int_1^{a} [x]\,f'(x)\,dx$$

$$I = \int_1^{2} 1.f'(x)\,dx + \int_2^{3} 2.f'(x)\,dx + \int_3^{4} 3.f'(x)\,dx + \ldots$$

$$+ \int_{[a]}^{a} [a].f'(x)\,dx$$

= [f(2) – f(1)] + 2 {f(3) – f(2)} + 3 (f(4) – f(9)) + ...
+ [a] (f(a) – f[a]
= –f(1) – f(2) – f(3) – f(4) ... f(a) + [a] + (a)
= [a] f(a) – {f(1) + f(2) + f(3) ..... – f [a]}

Sol.48 A

$$\lim_{x\to 2} \frac{\int_6^{f(x)} 4t^3}{x-2}\,dt$$

Apply L′Hospital

$$\lim_{x\to 2} \frac{4f^3(x).f'(x)}{1}$$

$$4f^3(2)\,.\,f^1(2) = 4 \times 6^3 \times \frac{1}{48} = 18$$

Sol.49 C

In (0, 1)

$x^2 > x^3$

$2^{x^2} > 2^{x^3}$

$$\int_0^1 2^{x^2}\,dx > \int_0^1 2^{x^3}\,dx$$

$I_1 = I_2$

Sol.50 C

$$I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{|\sin x + \cos x|}\,dx = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{(\sin x + \cos x)}\,dx$$

$$= \int_0^{\pi/2} (\sin x + \cos x)\,dx \quad [\sin x - \cos x]_0^{\pi/2} = 1 + 1 = 2$$

Sol.51 A

$$f(a) + f(-a) = \frac{e^a}{1+e^a} + \frac{e^{-a}}{1+e^{-a}} = 1$$

$$I_1 = \int_{f(-a)}^{f(a)} xg[x(1-x)]\,dx$$

Apply king

$$I_1 = \int_{f(-a)}^{f(a)} (1-x)g\{(1-x)x\}\,dx$$

$$2I_1 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}\,dx$$

$2I_1 = I_2$

$$\frac{I_2}{I_1} = 2$$

Sol.52 B

$$f(t) = \int_0^{t} f(t-2)g(g)dy = \int_0^{t} e^{(t-y)}.y\,dy$$

$$= e^t \int_0^{t} \underset{\text{II}}{\underset{\downarrow}{e^{-x}}}.\underset{\text{I}}{\underset{\downarrow}{x}}\,dx = e^t\left[-x e^{-x} + \int e^{-x}dx\right]$$

$$= e^t [-x e^{-x} + e^{-x}]_0^t = e^t [-t e^{-t} - e^{-t} + 1] = e^t - (t + 1)$$

Sol.53 B

$f(a + b - x) = f(x)$

$$I = \int_a^{a} x f(x)\,dx$$

King

$$I = \int_a^{b} (a+b-x) + (a + b - x)\,dx$$

$$I = \int_a^b (a+b-x)\,f(n)\,dx$$

$$2I = (a+b)\int_a^b f(x)\,dx$$

Sol.54 C

$$L = \lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 + dt}{x \sin x} = \lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\,dt}{x^2}$$

$$= \lim_{x\to 0} \frac{\sec^2 x^2 .2x}{2x} = \sec^2 (0) = 1$$

Sol.55 C

$$I = \int_a^1 x(1-x)^n dx$$

Alitar

Use king properly

Put $1 - x = t$

$dx = - dt$

$$= -\int_1^0 (1-x)t^n dt = \int_0^1 (1-x)t^n dt$$

$$= \int_0^1 (t^n - t^{n+1})dt = \frac{t^{n+1}}{n+1} - \frac{t^{n+2}}{n+2}\bigg|_0^1 = \frac{1}{n+1} - \frac{1}{n+2}$$

Sol.56 D

$$\int_1^4 \frac{3}{x} e^{\sin x^3}\, dx = \int_1^4 \frac{3x^2}{x^3} e^{\sin x^3} dx$$

$x^3 = t$

$$3x^2\, dx = dt = \int_1^{64} \frac{e^{\sin t}}{t} dt = f(t)\big|_1^{64} = f(64) - f(1)$$

Sol.57 C

$$\because\ f'(x) = f(x) \Rightarrow \frac{f'(x)}{f(x)} = 1$$

$$\int \frac{f'(x)}{f(x)}\, dx = \int dx \Rightarrow \ell n\, f(x) = x + c$$

$$f(x) = e^{x + c} \quad .....(1)$$

$f(0) = 1 \Rightarrow e^c = 1 \Rightarrow c = 0$

Now $f(x) = e^x$

$f(x) + g(x) = x^2 \Rightarrow g(x) = x^2 - e^x$

$$I = \int_0^1 f(x)\, g(x)\, dx = \int_0^1 e^x \{x^2 - e^x\}\, dx$$

$$= e - \frac{e^2}{2} - \frac{3}{2}$$

Sol.58 D

$$I = \int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$$

Applying King

$$I = \int_0^{\pi/2} \frac{dx}{1+\cot^3 x}$$

Add

$$2I = \int_0^{\pi/2} dx$$

$$I = \frac{\pi}{4}$$

Sol.59 A

$$\int_{\sin x}^1 t^2 f(t)\, dt = 1 - \sin x$$

Applying Leibitz Rule

$- \sin^2 x\ f(\sin t)\cos t = - \cos t$

$$f(\sec x) = \frac{1}{\sin^2 x}$$

$$f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)^2} = 3$$

$$\int_{\sin x}^1 t^2 + \underset{\substack{\downarrow \\ 1}}{f}t\, dt = 1 - \sin x$$

$$f(t) = \frac{1}{t^2}$$

$$f\left(\frac{1}{\sqrt{3}}\right) = 3$$

Sol.60 A

$$I_n = \int_0^{\pi/4} \tan^2 x dx$$

$$\frac{1}{I_2 + I_4} = \frac{1}{\int_6^{\pi/4} (\tan^2 x + \tan^4 x)\,dx}$$

$$= \frac{1}{\int_6^{\pi/4} \tan^2 x + \sin^2 x\,dx} = 3$$

$$\frac{1}{I_3 + I_5} = 4$$

$$\frac{1}{I_4 + I_6} = 5 \qquad \text{A.P.}$$

Sol.61 B

$$I = \lim_{x\to 0} \frac{\int_0^{x^2} \cos t^2\,dt}{x \sin x} = \lim_{x\to 0} \frac{\int_0^{x^2} \cot^2 dt}{x^2}$$

$$= \lim_{x\to 0} \frac{\cos x^4 .2x}{2x} = 1$$

Sol.62 B

$$I = \int_0^{\pi/4} \sin(x - [x])\,dx(x - [x])$$

$$0 < x < \frac{\pi}{4}$$

$[x] = 0$

$$= \int_1^3 [3 - x]dx + \int_3^5 [x - 3]dx$$

$3 - x = t$ $\qquad$ $x - 3 = 4$
$dx = - dt$ $\qquad$ $dx = du$

$$= -\int_2^0 [t]\,dt + \int_2^2 [u]\,du$$

$$= 2\int_0^2 [t]\,dt = 2\left[\int_0^1 0.dt + \int_0^2 0.dt\right] = 2$$

Sol.63 B

$$I = \int_1^5 [\,|x - 3|\,]\,dx$$

$$= \int_{-2}^2 [\,|x|\,]\,dx = 2\int_0^2 [\,|x|\,]\,dx$$

$$= 2\int_0^2 [x]\,dx = 2[1] = 2$$

Sol.64 A

$$I = \int_{-1}^3 \left(\tan^{-1}\frac{x}{1+x^2} + \tan^{-1}\frac{x^2+1}{x}\right)dx$$

$$I = \int_{-1}^1 \left(\tan^{-1}\frac{x}{1+x^2} + \tan^{-1}\frac{x^2+1}{x}\right)dx$$

O is odd function

$$+ \int_1^3 \left(\tan^{-1}\frac{x}{1+x^2} + \tan^{-1}\frac{x^2+1}{x}\right)dx$$

$$= \int_1^3 \left(\tan^{-1}\frac{x}{1+x^2} + \cot^{-1}\frac{x}{1+x^2}\right)dx = \int_1^3 \frac{\pi}{2}.dx = \frac{\pi}{2}(z) = \pi$$

Sol.65 A

$$I = \int_0^1 c_2x^2 + c_1x + c_0 = \frac{c_2x^2}{3} + \frac{c_1x^2}{2} + c_0x\Big|_0^1$$

$$= \frac{c_2}{3} + \frac{c_1}{2} + c_0$$

$I = 0$ $\quad$ than definately one root will lie in (0, 1)

Sol.66 A

$$I = \int_1^2 f'(x)\,dx = f(z) - f(1) = 0$$

Sol.67 A

$$\int_1^2 (x - \log_x a)\,dx = 2\log_2\left(\frac{2}{a}\right)$$

$$\frac{x^2}{2} - \log_z a \neq \Big|_a^2 = 2\log_2\left(\frac{2}{a}\right)$$

$2 - 2\log_2 a = 2\log_2 \dfrac{2}{a}$

$2 - 2\log_2 a = 2\log_2 2 - 2\log_2 a$

$1 = 1$

$a > 0$ because of log properties.

Sol.68 C

$$I = \int_{-1}^{1} \frac{x^4}{1+e^{x^7}}\,dx$$

King Replace $x \to \lambda - x - x + (x - 1 - x)$

$$I = \int_{-1}^{1} \frac{x^4}{1+e^{-x^7}}\,dx$$

$$2I = \int_{-1}^{1} \frac{x^4(1+e^{x^7})}{(1+e^{x^7})}\,dx$$

$$I = \frac{1}{5}$$

Sol.69 B

$$I = \frac{1}{C} = \int_{ac}^{bc} f\left(\frac{x}{c}\right)dx$$

Put $\dfrac{x}{c} = t \Rightarrow \dfrac{dx}{c} = dt \quad = \displaystyle\int_a^b d(t)\,dt$

Sol.70 C

$$\int_{\ell n2}^{x} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$$

Put $e^x - 1 = t^2$

$e^x\,dx = 2t\,dt$

$$dx = \frac{2t\,dt}{(t^2+1)}$$

$$2\int_1^{\sqrt{e^x-1}} \frac{dt}{t^2+1} = \frac{\pi}{6}$$

$2\tan^{-1}\sqrt{e^x - 1} = 30^o + 90^o$

$\tan^{-1}\sqrt{e^x - 1} = 60^o$

$\sqrt{e^x - 1} = \sqrt{3}$

$e^x - 1 = 3$

$e^x = 4 \qquad \Rightarrow x = \ell n\, 4$

Sol.71 C

$$I_1 = \int_0^1 \frac{e^x dx}{1+x}$$ \

$$I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2 - x^3)}\,dx$$

Put $x^3 = t$

$$x^2\,dx = \frac{dt}{3} = \frac{1}{3}\int_0^1 \frac{dt}{e^t(2-5)}$$

$1 - t = z$

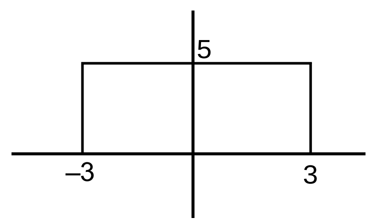
$dt = -dz$

$$= -\frac{1}{3}\int_1^0 \frac{dz}{e^{(1-z)}(1+z)} = \frac{1}{3}\int_0^1 \frac{e^x dz}{e^{(1-z)}(1+z)}$$

$$I_2 = \frac{1}{3e} I_1$$

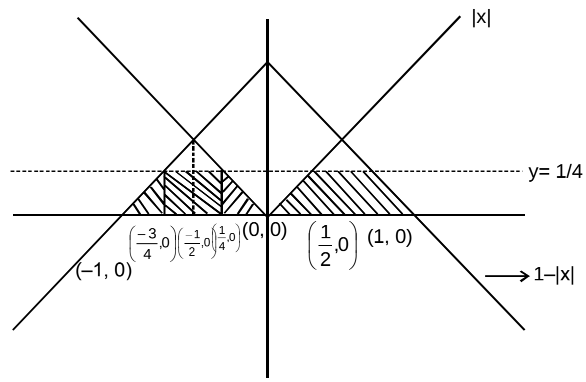
$$\frac{I_1}{I_2} = 3e$$

Sol.72 D



$$\int_{-3}^{3} f(x)\,dx = 5 \times (3 + 3) = 30$$

Sol.73 B



$$\int_{-1}^{1} f(x)\,dx = 2\,[A_1 + A_2 + A_3] = 2$$

$$[\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{4}\right) + \left(\frac{1}{2}\times\frac{1}{4}\right) + \left(\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}\right)] = \frac{3}{8}$$

Sol.74 A

$$I = \int_{-\pi/4}^{\pi/4} \underbrace{\underbrace{\left\{\frac{e^x}{e^{2x}-1}\right\}}_{\text{odd fn.}} \underbrace{\sec^2 x}_{\text{Even Function}} dx}_{\text{Odd function}}$$

$I = 0$

Sol.75 B

$$f(x) = \int_0^x (t^2 - t + 1)\,dt \quad \forall\ x \in (3, 4)$$

$$\text{Geatest} = \int_0^4 (t^2 - t + 1)\,dt$$

$$\text{Least} = \int_0^3 (t^2 - t + 1)\,dt$$

$$\text{diff.} = \int_0^4 (t^2 - t + 1)\,dt - \int_0^3 (t^2 - t + 1)\,dt$$

$$= \int_3^4 (t^2 - t + 1)\,dt = \left.\frac{t^3}{3} - \frac{t^3}{2} + t\right|_3^4 = \frac{59}{6}$$

Sol.76 B

$$0 < x < \frac{\pi}{2}$$

$$I = \int_{1/\sqrt{2}}^{1/2} \cot x \, d(\cos x)$$

$$= -\int_{\pi/4}^{\pi/3} \frac{\cos x}{\sin x} \sin x \, dx = -\ [\sin x]_{\pi/4}^{\pi/3}$$

$$= -\left[\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right] = -\left(\frac{\sqrt{3}-\sqrt{2}}{2}\right) = \frac{\sqrt{2}-\sqrt{3}}{2}$$

Sol.77 C

$$f(x) = \begin{cases} e^{\cos x} \sin x & , \quad -2 \le x \le 2 \\ 2 & , \quad \text{otherwise} \end{cases}$$

$$I = \int_{-2}^{3} f(x)\,dx + \int_{2}^{3} f(x)\,dx$$

$$= \int_{-2}^{2} e^{\cos x} \underbrace{\sin x}_{\substack{\text{O as} \\ \text{odd} \\ \text{funtion}}}\, dx + \int_2^3 2.dx = 2$$

Sol.78 C

$$I = \int_0^{\pi/3} [\sqrt{3}\tan x]\,dx$$

$\sqrt{3}\ \tan x = t$

$\sqrt{3}\ \sec^2 x\ dx = dt$

$$dx = \frac{dt}{\sqrt{3}(1+\tan^2 x)} = \frac{dt}{\sqrt{3}\left(1+\frac{t^2}{3}\right)} = \frac{\sqrt{3}dt}{t^2+3}$$

$$= \int_0^3 [t] . \frac{\sqrt{3}dt}{t^2+3}$$

$$= \sqrt{3} \int_0^1 \frac{0.dt}{t^2+3} + \sqrt{3} \int_1^2 \frac{dt}{t^2+3} + 2\sqrt{3} \int_0^3 \frac{dt}{t^2+3}$$

$$= \tan^{-1} \left.\frac{t}{\sqrt{3}}\right|_1^2 + 2\tan^{-1} \left.\frac{t}{\sqrt{3}}\right|_2^3$$

$$= \tan^{-1}\frac{2}{\sqrt{3}} - \frac{\pi}{6} + 2\tan^{-1}\sqrt{3} - 2\tan^{-1}\frac{2}{\sqrt{3}}$$

$$= \frac{2\pi}{3} - \frac{\pi}{6} - \tan^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{2} - \tan^{-1}\frac{2}{\sqrt{3}}$$

Sol.79 C

$$I = \int_{-1}^{1} \frac{\sin x + x^2}{3-|x|}\,dx$$

$$= \underbrace{\int_{-1}^{1} \frac{\sin x + x^2}{3-|x|}}_{\text{O as odd Function}} + \int_{-1}^{1} \frac{x^2}{3-|x|}dx = 2\int_0^1 \frac{x^2}{3-|x|}dx$$

Sol.80 B

$I_1 = \int_1^2 \frac{dx}{1+x^2}$ $\quad I_2 = \int_1^2 \frac{dx}{x}$

$= \ell n\,(x+\sqrt{x^2+1})\,|_1^2 = \ell n\{2+\sqrt{5}\} - \ell n\{1+\sqrt{2}\}$

$= \ell n\left(\frac{2+\sqrt{3}}{1+\sqrt{2}}\right) = \ell n\left(\frac{2+2.3}{1+1.4}\right) = \ell n\left(\frac{4.3}{2.4}\right) = \ell n\,(1.8)$

$I_2 > I_1$

$I_2 = \ell n 2$

Alitar $\quad x^2 + 1 > x^2$

$$\frac{1}{x^2} > \frac{1}{x^2+1} \Rightarrow \frac{1}{x} > \frac{1}{\sqrt{x^2+1}}$$

$$\int_1^2 \frac{1}{x}dx > \int_1^2 \frac{dx}{\sqrt{x^2+1}}$$

$I_2 > I_1$

Sol.81 A

$I = \int_{5/2}^{5} \frac{\sqrt{(25-x^2)^3}}{x^4}dx$

Put $x = 5\sin\theta \Rightarrow \quad dx = 5\cos\theta\, d\theta$

$= \int_{\pi/6}^{\pi/2} \cot^4\theta\, d\theta = \int_{\pi/6}^{\pi/2} \cot^2\theta\,(\operatorname{cosec}^2\theta - 1)\, d\theta$

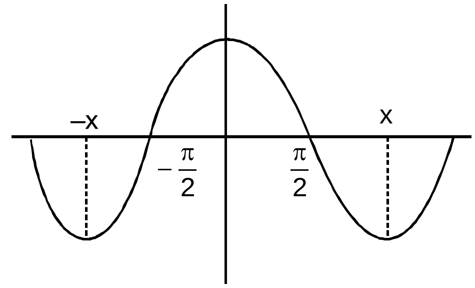
$= \int \cot^2\theta \operatorname{cosec}^2\theta d\theta - \int \cot^2\theta\, d\theta$

$= \left.\frac{\cot^2 3\theta}{3}\right|_{\pi/6}^{\pi/2} - \int \operatorname{cosec}^2\theta + \theta$

$= -\frac{\cot^3\theta}{3} + \cot\theta + \theta\,|_{\pi/6}^{\pi/2}$

$= \frac{\pi}{2} + \frac{(\sqrt{3})^3}{3} - \sqrt{3} - \frac{\pi}{6} = \frac{\pi}{3}$

Sol.82 C



$I = \int_{-2}^{-1} \left[x\left[1+\cos\left(\frac{\pi x}{2}\right)\right] + 1\right] dx$

$-2x < x < 1$

$-p < \frac{\pi x}{2} < \frac{\pi}{2}$

$-2 < x < -1$

$-\pi < \frac{\pi x}{2} < 0$

$-1 < \cos\frac{\pi x}{2} < 0$

$0 < 1 + \cos\frac{\pi x}{2} < 1$

$\left[1+\cos\frac{\pi x}{2}\right] = 0$

$\int_{-2}^{-1} 1.dx + \int_{-1}^{0} [x+1]dx + \int_{0}^{1} [x+1]dx$

$-\frac{\pi}{2} < \frac{\pi x}{2} < 0 \quad 0 < \frac{\pi x}{2} < \frac{\pi}{2}$

$1 < \cos\frac{\pi x}{2} + 1 < 2 \qquad 1 < \cos\frac{\pi x}{2} + 1 < 2$

$= 1 + 0 + 1 = 2$

Sol.83 A

$I = \int_0^{[x]} [x]\, dx = [x]\int_0^1 \{x\}\, dx = [x]\int_0^1 x\, dx = \frac{[x]}{2}$

Sol.84 D

$$I = \int_0^1 e^{2x-[2x]}d(x-[x]) = \int_0^1 e^{[2x]}dx$$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

$$= \frac{1}{2}\int_0^2 e^{\{t\}}dt = \int_0^1 e^{\{t\}}dt = \int_0^1 e^{t}dt = e - 1$$

Sol.85 C

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2}$$

$$\lim_{n\to\infty}\sum_{r=0}^{n}\frac{\sqrt{n}/\sqrt{n}}{\frac{\sqrt{r}}{\sqrt{n}}n\left(4+3\sqrt{\frac{r}{n}}\right)^2}$$

$$\int_0^1 \frac{dx}{\sqrt{x}(4+3\sqrt{x})^2}$$

$r + 3\sqrt{x} = t$

$\frac{3}{2\sqrt{x}}\ dx = dt$

$$\frac{dx}{\sqrt{x}} = \frac{2}{3}dt = \frac{2}{3}\int\frac{dt}{t^2} = \frac{-2}{3}\ \frac{1}{t}$$

$$= -\frac{2}{3}\ \frac{1}{(4+3\sqrt{x})}\Bigg|_0^1 = \frac{-2}{3}\left[\frac{1}{n}-\frac{1}{4}\right]$$

$$= \frac{-2}{3}\left[\frac{4-7}{28}\right] = \frac{2}{28} = \frac{1}{14}$$

Sol.86 D

$$I = \int_{\pi/4}^{\pi/3}\operatorname{cosec} x\ d(\sin x)$$

$$= \int_{\sin^{-1}\frac{\pi}{4}}^{\sin^{-1}}\cot x\ dx = \ell n \sin x \quad \ell n\sin x\,\Big|_{\sin^{-1} n/4}^{\sin^{-1}\pi/3}$$

$$= \ell n \sin\left(\sin^{-1}\frac{\pi}{3}\right) - \ell n \sin\left(\sin^{-1}\frac{\pi}{4}\right)$$

$$= \ell n\ \frac{\pi}{3} - \ell n\ \frac{\pi}{4} = \ell n\frac{4}{3}$$

Sol.87 B

$$I = \int_0^2 x^3\left[1+\cos\frac{\pi x}{2}\right]dx$$

$$= \int_0^1 x^3\left[1+\cos\frac{\pi x}{2}\right]dx + \int_0^2 x^3\left[1+\cos\frac{\pi x}{2}\right]dx$$

$0 < x < 1$	$1 < x < 2$
$0 < \frac{\pi}{2}x < \frac{\pi}{2}$	$\frac{\pi}{2} < \frac{\pi}{2}\ x < \pi$
$1 > \cos\ \frac{\pi}{2}\ x > 0$	$0 > \cos\ \frac{\pi}{2}\ x > -1$
$1 < 1 + \cos\ \frac{\pi}{2}\ x > 0$	$0 < 1 + \cos\ \frac{\pi}{2}\ x < 1$
$[1 + \cos\ \frac{\pi}{2}x] = 1$	$[1 + \cos\ \frac{\pi}{2}x] = 0$

$$= \int_0^1 x^3 dx + \int_1^2 x^3(0)dx = \frac{1}{4}$$